Corks, plugs, Glucks, and ropes

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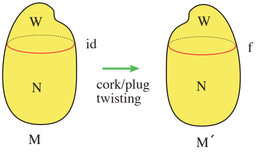
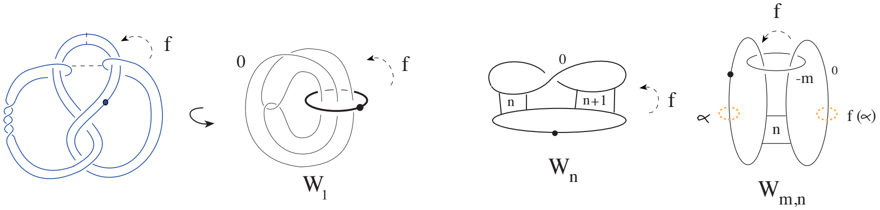


- cork ≃ point (A, Matveyev [Curtis, Freedman, Hsiang, Stong])
- plug ≃ S^2 (A and Yasui)
- "anticork" ≃ S^1

Examples: W_n corks, $W_{m,n}$ plugs ($m \geq 1, n \geq 2$). $W_{1,0}$ is Gluck!



Remarks about plugs

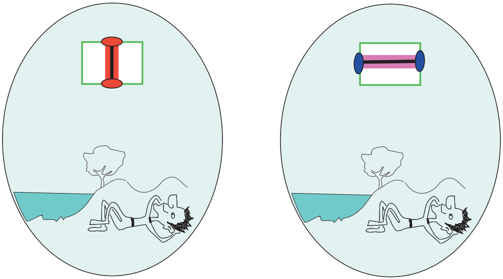
- $W_{m,n}$ is a plug: If f extended homemorphism inside $\Longrightarrow$

$$W_{m,n} + \gamma^{-1} \approx W_{m,n} + f(\gamma)^{-1}$$

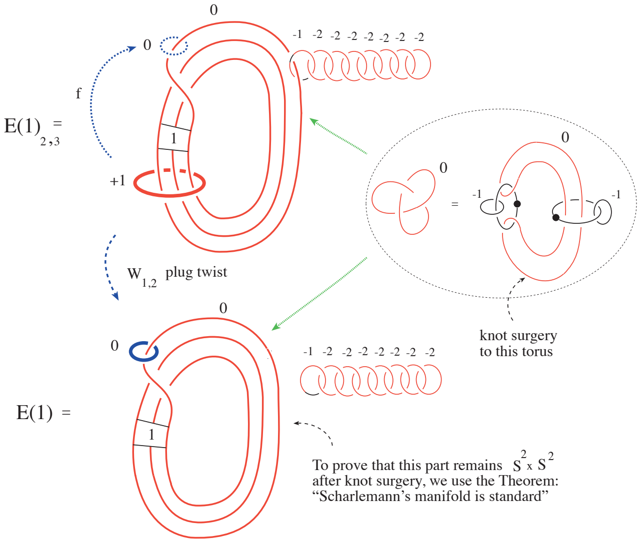
but they have nonisomorphic intersection forms

$$\begin{pmatrix} -2n - mn^2 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} -2n - mn^2 & -1 - mn \\ -1 - mn & -1 - m \end{pmatrix},$$

- $W_{m,n} = B^4 + K^{-2n - n^2 m^2}$, $K \subset S^3$ is some torus knot. So upside plug also looks like a 2-handle attached to different place.

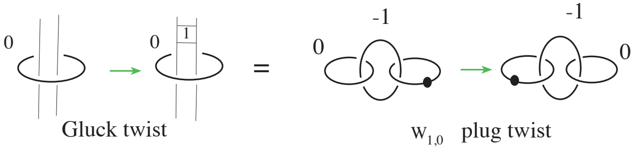


Seeing standard and exotic copies simultaneously

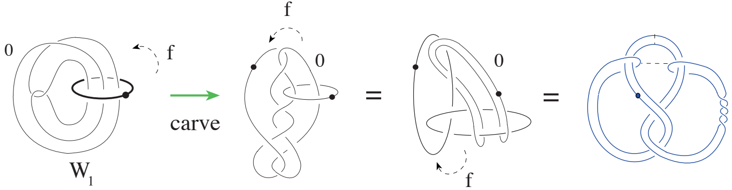


More applications

- Gluck twisting $X^4 \mapsto X_S$, where $S \subset X^2$ smooth 2-sphere. Gluck-twisting = $W_{1,0}$ plug-twisting.



- Theorem (A-Yasui) If $S \subset X^4$ is null homologous, and the intersection form of X is odd then $X_S \approx X$
- Creating an anticork:



Tools:

$$
\begin{array}{ccc}
\mathcal{E} = \{\text{Eliashberg Handlebodies}\} & \overset{A-O}{\twoheadleftarrow} & \{PALF's\} = \mathcal{P} \\
\downarrow \text{E} & \swarrow & \downarrow \text{A-O} \\
\{\text{Stein Manifolds}\} & \dashrightarrow & \{\text{Symplectic manifolds}\}
\end{array}
$$

- $X^4 \in \mathcal{E}$, choose $\mathcal{F}$ with $X^4 = |\mathcal{F}| \Rightarrow X^4 \subset S^4$ (closed symplectic)
  $X \in \mathcal{E} \Rightarrow X = \{K_1^{r_1}, ...K_s^{r_s}, C_1, .., C_p\}$ (1-and 2-handles)
  Let $K_S \to S$ can. line bundle, $< c_1(M), K_j >= rot(K_j)$, K_j^* hom duals

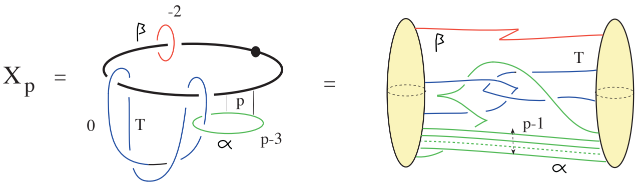
$$K_S = c_1(S) = \sum_j rot(K_j)K_j^*$$

- $\Sigma \subset S \quad \Rightarrow \quad 2g - 2 \geq \Sigma.\Sigma + |K_S.\Sigma|$ (Adjunction inequality)

Examples:

Theorem (A-Yasui) ∃ *infinitely many* 1*-connected Stein manifolds (of Betti number 2), which are exotic copies of each other. They are Stein fillings of the same contact* 3*-manifold*.

(Existence of such examples, with large Betti numbers, were previously established by Akhmedov-Etnyre-Mark-Smith)



proof: Apply adjunction to $\Sigma = \alpha - p\beta + qT$, where $q = p^2 - (p-3)/2$

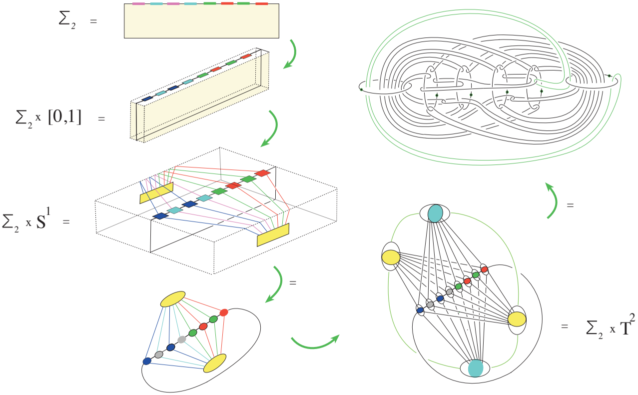
$$2g(\Sigma) - 2 \geq \Sigma.\Sigma + |K_S.\Sigma| = 0 + |p-1|$$

$$\Rightarrow g(\Sigma) \to \infty \text{ as } p \to \infty$$

2 recent examples

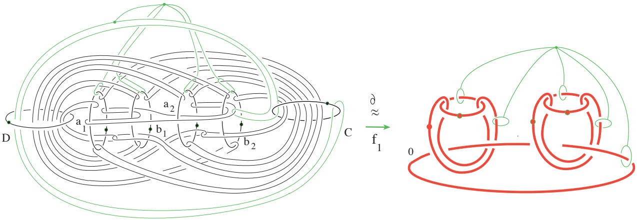
- Problem: How to cut out a big a chunk from a handlebody, and putting a different chunk insead with the same boundary?
- Solution: "Ropes technique". We will demonstrate this on the Akhmedov-Park manifolds.
- Fibration $T^2 \to S^3_0(K) \times S^1 \to T^2$.
- AP1: Smooth $T^2_{vert} \cup T^2_{hor} \subset S^3_0(K) \times S^1$, then blow up $\Rightarrow$ Get $F_2 \times D^2 \subset S^3_0(K) \times S^1 \# 2(-\mathbb{CP}^2) \Rightarrow$ Call its omplement X
- AP2: Blowup $S^3_0(K) \times S^1 \# (-\mathbb{CP}^2)$, then smooth $2T^2_{vert} \cup T^2_{hor} \Rightarrow$ $F_2 \times D^2 \subset S^3_0(K) \times S^1 \# (-\mathbb{CP}^2) \Rightarrow$ Call its omplement Y
- $Z = F_2 \times T^2_0$ with 4 Luttinger surgeries in its interior.
- AP1: $M = Z \smile X$
- AP2: $N = Z \smile Y$

Building $\Sigma_2 \times T^2$



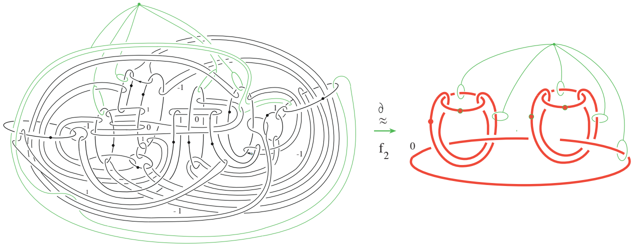
Lowering ropes

(1) Draw $F_2 \times T^2_0$ with a concrete diffeo $f_1 : \partial(F_2 \times T^2_0) \overset{\approx}{\to} \partial(F_2 \times D^2)$

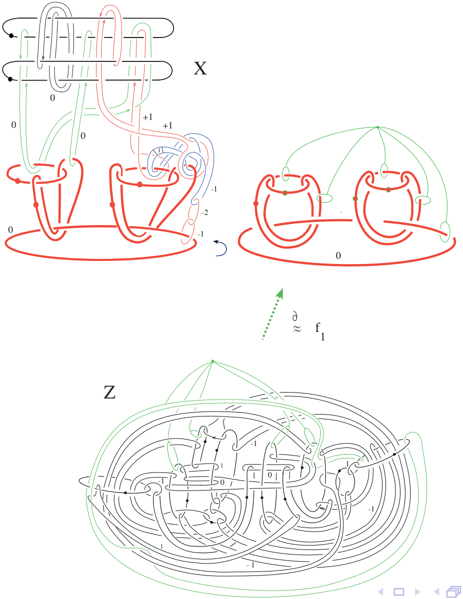


- (2) Draw Z with a concrete diffeo $f_1 : \partial Z \overset{\approx}{\to} \partial(F_2 \times D^2)$

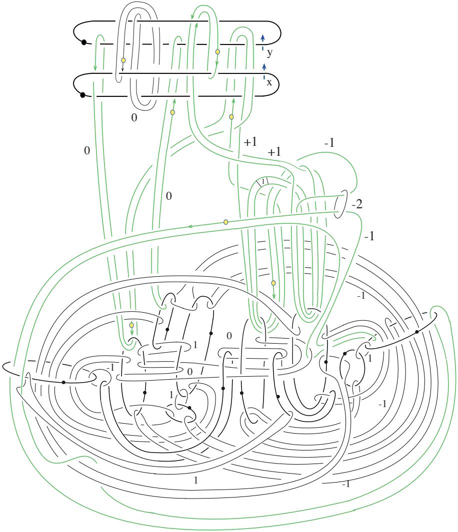
Z =



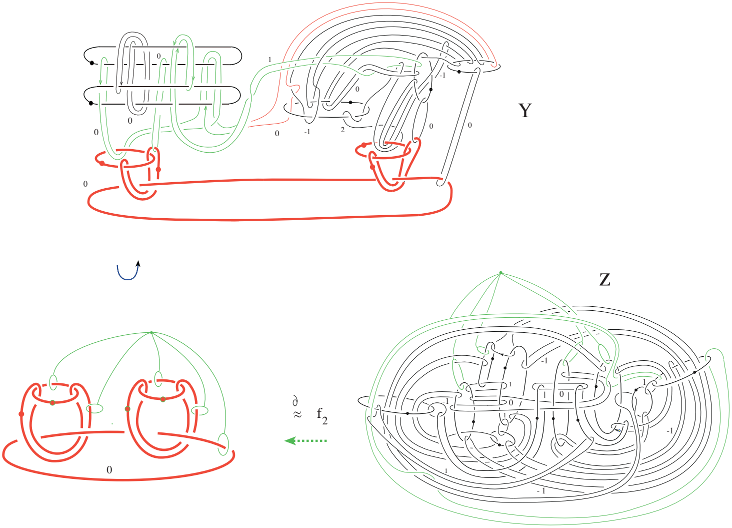
Ropes



M



Ropes



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