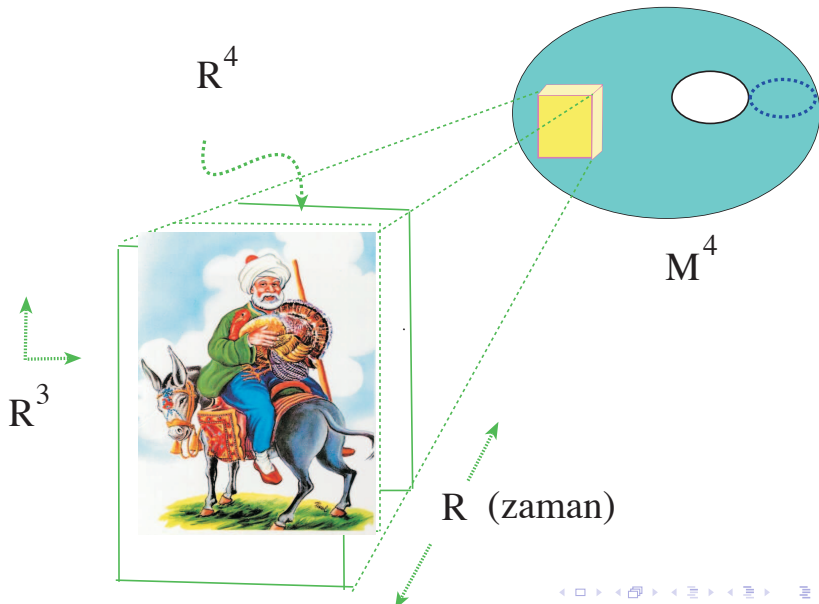


4-boyutda uzay insaati

Selman Akbulut

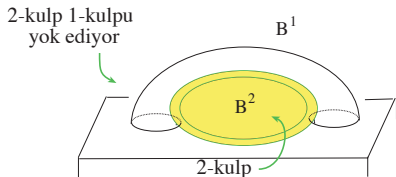
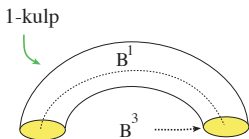
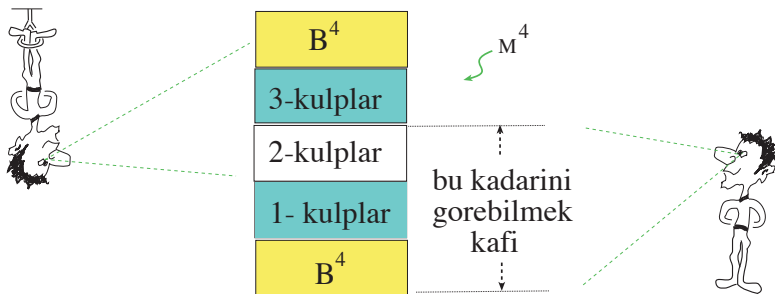
September 5, 2011

4-boyutlu uzay (manifold)



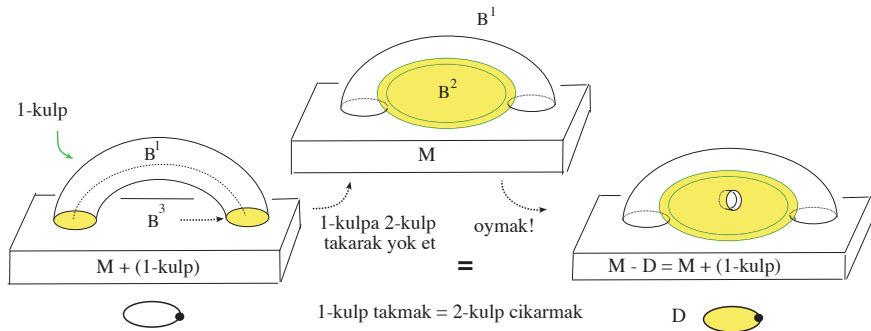
4-manifold'un kulplarla tasviri (Morse fonksiyonu)

k-kulp: $B^4 = \mathbf{B}^k \times B^{4-k}$ ($k = 0, 1, 2, 3, 4$) kenari olan
 $\partial \mathbf{B}^k \times B^{4-k} = \mathbf{S}^{k-1} \times B^{4-k}$ den diger kulplara yapistiriliyor.

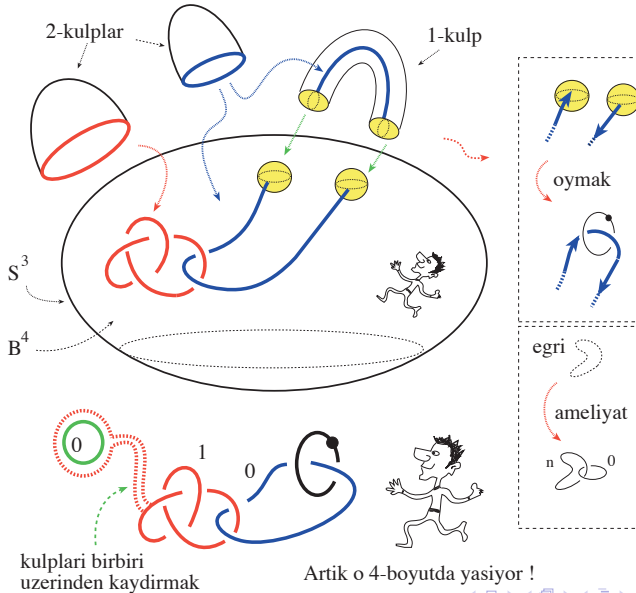


4-manifold kulplarının riayet ettiği 3-temel prensip!

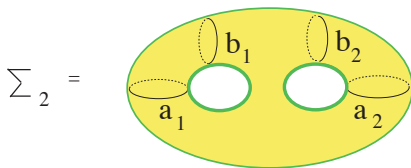
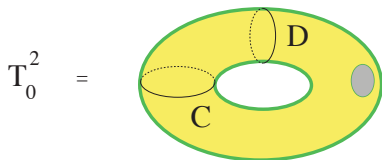
- (1) 0, 1, and 2-kulplarını görmek kafi (Laudenbach-Poenaru/1972)
- (2) "**Vermek=Almak**" : 1-kulp yapistirmek = 2-kulp cikarmak (A/1977)
- (3) Karisik 4-manifoldların kulplarına tersinden bakarsan basitlesir !!



Simdi 4-manifoldlari hayal edelim



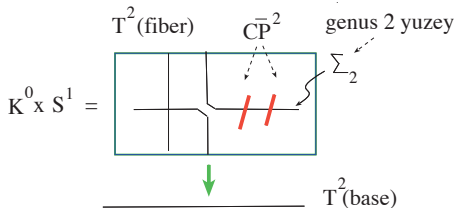
Torus T^2 ve Genus iki yuzey Σ_2



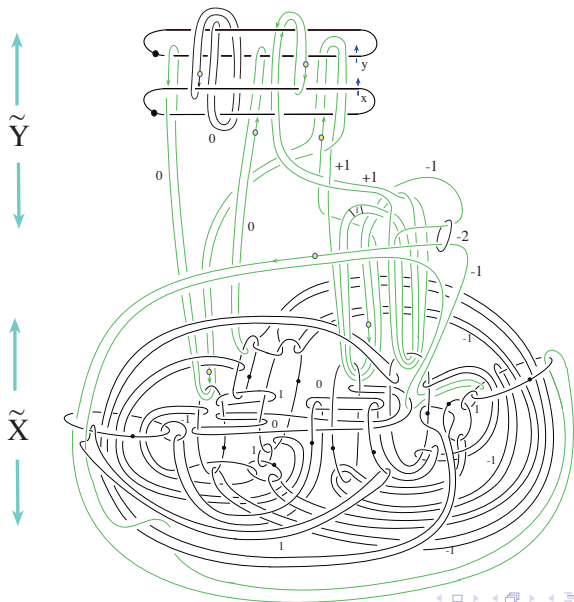
$$M^4 = \Sigma_2 \times T_0^2$$

$$M = \tilde{X} \cup_{\partial} \tilde{Y}$$

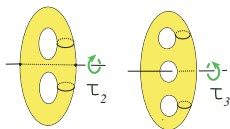
- $\Sigma_2 =$ genus 2 yuzey $\langle a_1, b_1, a_2, b_2 \rangle$
- $T_0^2 =$ 1- delikli torus $\langle C, D \rangle$
- $X = \Sigma_2 \times T_0^2$
- $\tilde{X} =$ Log transform $X \langle a_1 \times C, b_1 \times C, a_2 \times C, a_2 \times D \rangle$
- $Y = (K^0 \times S^1) \# 2\bar{CP}^2$ (trefoil dugumu $K \subset S^3$)
- $\tilde{Y} = Y - \Sigma_2 \times D^2$



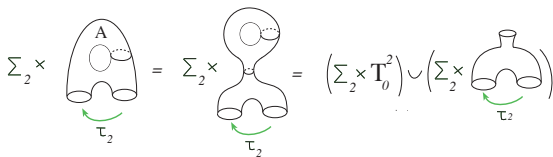
Akhmedov-Park ekzotik $CP^2 \# 3\bar{CP}^2$ (resim)



Cacime complex yuzeyi (tarifi)



$$X = (\Sigma_2 \times \Sigma_3) / \tau_2 \times \tau_3 \longrightarrow \Sigma_2 \quad (\Sigma_2 \text{ uzerinde } \Sigma_2 \text{-bundle})$$



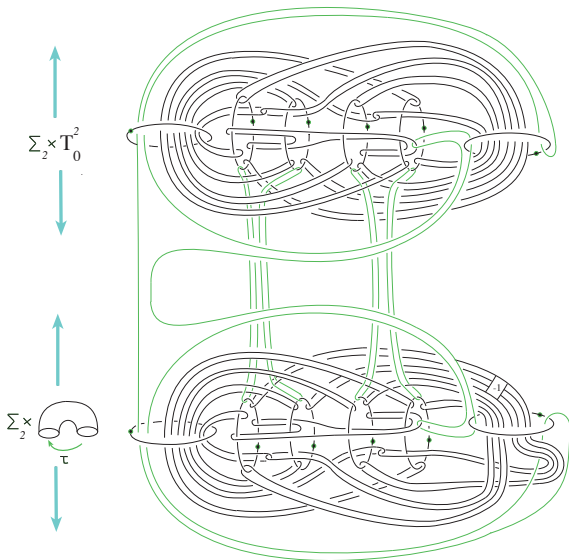
Theorem

(Hacon-Pardini/Pirola): X complex yuzey olsun oyle ki $p_g(X) = q(X) = 3 \Rightarrow X$ asagidaki yuzeylerden biridir:

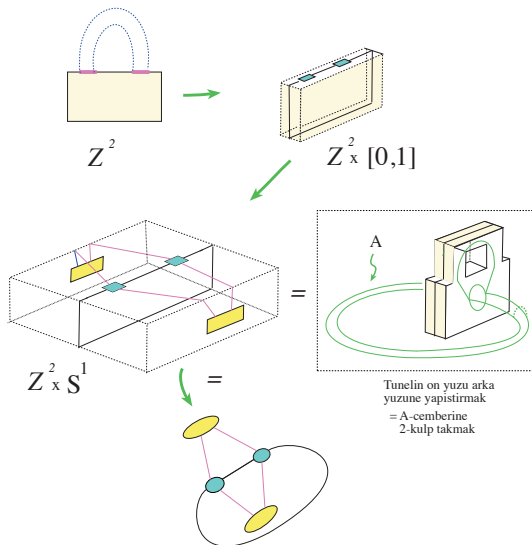
(a) $K^2 = 6 \Rightarrow X = \text{Sym}^2(\Sigma_3)$

(b) $K^2 = 8 \Rightarrow X = \text{Cacime yuzeyi}$.

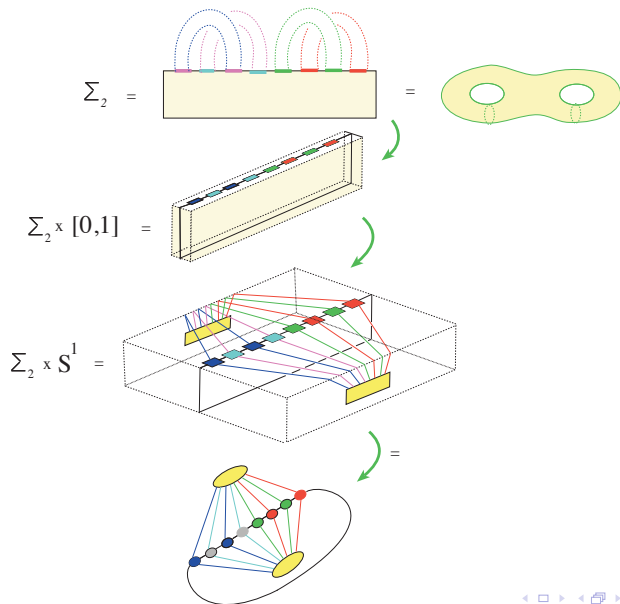
Cacime complex yuzeyi (resmi)



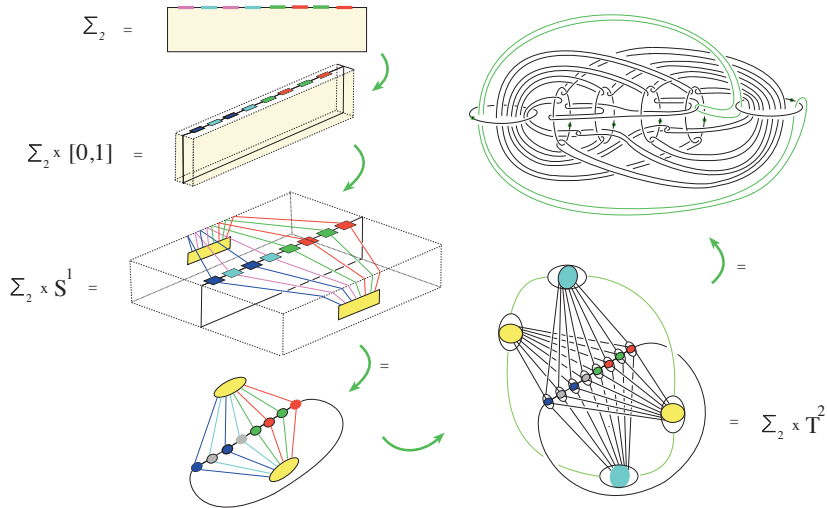
$Z^2 \times S^1$ resmi



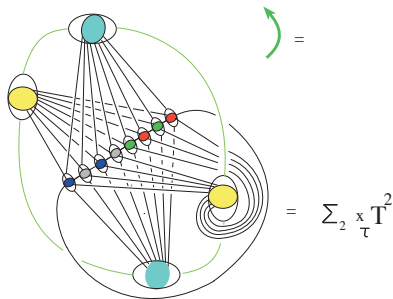
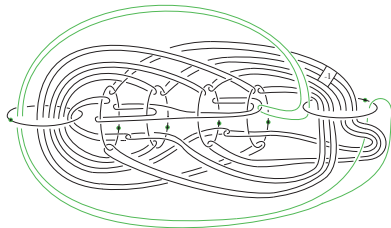
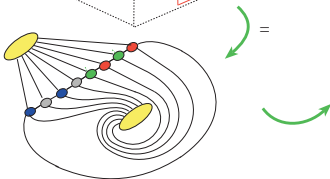
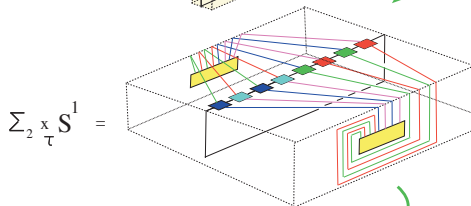
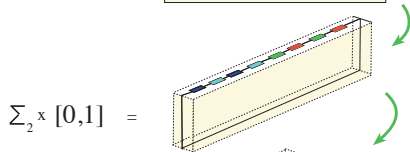
$\Sigma_2 \times S^1$ resmi



$\Sigma_2 \times T^2 \rightarrow T^2$ resmi



$\Sigma_2 \times_{\tau} T^2 \rightarrow T^2$ resmi



Yuzeylerini $f_2 \circ f_1^{-1}$ ile yapistir. Metod: egrileri takip et!

